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A SIMPLE PROOF OF A THEOREM WITH REFERENCE TO TANGENTS TOUCHING A SURFACE IN TWO POINTS.

By Prof. James H. Boyd, St. Paul, Minn.

Through any point on a surface can be drawn $(n+2)\,(n-3)$ tangents which will also touch the surface elsewhere, n being the degree of the surface. (*Vide* Salmon's Geometry of Three Dimensions, Art. 275.)

Let P be the point of tangency on the surface. All the tangents to the surface at the point P lie in the same plane, tangent to the surface at P. Then all the tangents at P which touch the surface again will touch it in the curve Q which the tangent plane at P cuts out of the surface.

The problem is now reduced to finding how many tangents can be drawn from P to the curve Q. The curve Q will be of the nth degree, and the point P will be a triple point, because every plane tangent to a surface passes through three coincident points; *this point* is a point on the curve Q, hence triple.

From P to the curve Q there can be drawn $n\,(n-1)$ tangent lines. (*Vide* Salmon's Higher Plane Curves, Art. 65.) But since P is a triple point on the curve Q, the number of tangents is reduced by six. (*Vide* Salmon's Higher Plane Curves, Art. 68.)

Hence the number of tangents that can be drawn from P to the curve Q is $n\,(n-1)-6$, i. e. $(n+2)\,(n-3)$. Q. E. D.

THE BITANGENTS OF THE QUINTIC.

(Letter from Professor Cayley to Mr. Heal.)

Dear Sir: I have to thank you very much for your letter concerning the bitangents of the quintic, and am glad you have obtained more simple numerical coefficients than you had at first. I do not see my way to verifying your result, but assuming it to be correct, it is a very interesting and valuable one. The relation to Salmon's Higher Plane Curves (p. 351) is rather curious, viz., the result there given is of the deg-order 18-54, with an extraneous factor $(\alpha x+\beta y+\gamma z)^6$, the rejection of which would reduce it to the proper form, deg-order 18-48; whereas yours is of deg-order 24-66, with extraneous factor H^2, the rejection of which would reduce it to the same proper form, deg-order 18-48.

But there is another known solution, not by any means a handy one, but which has no extraneous factor,* viz: if

$$\Omega = D^3 H - 4\, D^3 H_1 + 6\, D^3 H_2,$$

then the required equation is

$$F\,\Omega = 0,$$

where the facients of the reciprocant are the first derived functions,

$$\partial_x U,\ \partial_y U,\ \partial_z U.$$

Say this is

$$(B^2 C^2 - \ldots)\,(\partial_x U)^6 = 0;$$

the coefficients A, B, C, etc. are those of Ω, viz., these are of deg-order 3-6, and $\partial_x U$, etc., are of deg-order 1-4; so that the deg-order is

$$4\,(3\text{-}6) + 6\,(1\text{-}4) = (12\text{-}24) + (6\text{-}24) = (18\text{-}48),$$

as it should be. It might be worth while to further consider this form, but I doubt whether, practically, the whole question is not too difficult to be worth working at.

I remain, dear sir, yours very sincerely,

A. CAYLEY.

CAMBRIDGE, *Jan.* 17, 1890.

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* See Philosophical Transactions. Vol. CXLIX, pp. 193–212.